19.44. Visualize: Refer to Figure P19.44 in your textbook.

Solve: **(a)** Q_1 is given as 1000 J. Using the energy transfer equation for the heat engine,

$$Q_H = Q_C + W_{out} \Rightarrow Q_1 = Q_2 + W_{out} \Rightarrow Q_2 = Q_1 - W_{out}$$

The thermal efficiency of a Carnot engine is

$$\eta = 1 - \frac{T_C}{T_H} = 1 - \frac{300\text{ K}}{600\text{ K}} = 0.50 = \frac{W_{out}}{Q_1}$$

$$\Rightarrow Q_2 = Q_1 - \eta Q_1 = Q_1(1-\eta) = (1000\text{ J})(1-0.50) = 500\text{ J}$$

To determine Q_3 and Q_4, we turn our attention to the Carnot refrigerator, which is driven by the output of the heat engine with $W_{in} = W_{out}$. The coefficient of performance is

$$K = \frac{T_C}{T_H - T_C} = \frac{400\text{ K}}{500\text{ K} - 400\text{ K}} = 4.0 = \frac{Q_C}{W_{in}} = \frac{Q_4}{W_{out}} = \frac{Q_4}{\eta Q_1}$$

$$\Rightarrow Q_4 = K\eta Q_1 = (4.0)(0.50)(1000\text{ J}) = 2000\text{ J}$$

Using now the energy transfer equation $W_{in} + Q_4 = Q_3$, we have

$$Q_3 = W_{out} + Q_4 = \eta Q_1 + Q_4 = (0.50)(1000\text{ J}) + 2000\text{ J} = 2500\text{ J}$$

(b) From part (a) $Q_3 = 2500\text{ J}$ and $Q_1 = 1000\text{ J}$, so $Q_3 > Q_1$.

(c) Although $Q_1 = 1000$ J and $Q_3 = 2500$ J, the two devices together do not violate the second law of thermodynamics. This is because the hot and cold reservoirs are different for the heat engine and the refrigerator.